CSI 30 SPRING 24 PROF. PINEIRO FINAL EXAM PREPARATION

- 1. Write the propositions using simpler propositions and logical connectives and determine, if possible, true or false.
 - (a) If $4 \ge 1$ then x > 10.
 - (b) If x < 5 then $2 \le 25$
 - (c) If $3 \ge -2$ then 4 > 7.
 - (d) If 7 < 1 then 14 > 10
- 2. Show that the compound proposition $(p \land (p \rightarrow q)) \rightarrow q$ is a tautology using a truth table.
- 3. Consider the proposition 'Some people do go good in every situation'.
 - (a) Write the proposition using quantifiers. Express clearly the domain of your variables.
 - (b) Negate the proposition using quantifiers and express your answer in such a way that no quantifier is immediately preceded by a negation.
- 4. Let S be the set $S = \{\{1\}, \{2\}, 3\}$. Determine True or False for the following statements:
 - (a) $\{2\} \in S$.
 - (b) $\{2\} \subset S$.
 - (c) $\{3\} \subset S$.

5. Consider the sets $A = \{0, 1, 2\}$ and $B = \{a, b, c\}$.

- (a) List the elements in $\mathcal{P}(B)$.
- (b) Build $A \times B$.
- (c) Give an example of a function from A to B. Is your function one-to-one? onto?
- (d) Give an example of a relation from A to B that is not a function.
- 6. (5 points) Suppose that the universe $\mathbb{U} = \{\text{prime numbers } \le 30\}, A = \{7, 13, 29\}$ and $B = \{5, 11, 13, 17, 29\}.$
 - (a) Determine $A \cup B$ and $|A \cup B|$.
 - (b) Determine $A \cap B$ and $|A \cap B|$.
 - (c) Determine \overline{A} and $|\overline{A}|$.
 - (d) Determine A B.
 - (e) Represent A with a bit string of length 10 using in \mathbb{U} the increasing order.

7. (5 points) Given the algorithm:

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procedure partial(a_1, a_2, a_3, \dots, a_n): integers)

sum_1 := 0

sum_2 := 0

for i := 1 to n

if (a_i > 0): sum_1 := sum_1 + a_i

if (a_i < 0): sum_2 := sum_2 + a_i

return(sum_1, sum_2)
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For the set of values $\{-4, 5, -7, 2, 9, 0, -2\}$ as input for the above algorithm, what are values of $prod_1$ and $prod_2$ that will be returned?

8. Find the greatest common divisor GCD(578, 153) using the Euclidean Algorithm:

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procedure GCD (a, b: positive integers):
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\begin{array}{l} \mathbf{x} := \mathbf{a} \\ \mathbf{y} := \mathbf{b} \\ \text{While } y \neq \mathbf{0}: \\ \mathbf{r} := \mathbf{x} \mod \mathbf{y} \\ \mathbf{x} := \mathbf{y} \\ \mathbf{y} := \mathbf{r} \\ \text{return('The GCD is' : x)} \\ \text{Find integers } t, s \text{ such that } 578t + 153s = GCD(578, 153). \end{array}
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- 9. Consider the numbers n = 347 and $m = (1D3C)_{16}$.
 - (a) Find the representation of n is base 16.
 - (b) Compute the decimal representation of m. (Use $A = 10, B = 11, \ldots, F = 15$).
 - (c) Find m + n in base 16.
 - (d) Find m + n in base 10.
- 10. In how many ways can the letters of the word 'AYAYIYO' be arranged?
- 11. What is the coefficient of x^2y^4 in the expansion of the binomial $(3x 5y)^6$.
- 12. How many bit-strings of length 6 contain not consecutive zeroes?
- 13. How many integers from 1 to 100 are divisible by 2 or by 3?
- 14. Consider the following generator of pseudo-random numbers:

$$x_n = (5x_{n-1} + 7) \mod 12$$
, with seed $x_0 = 4$.

What sequence of pseudo-random numbers does it generate?

15. What is the probability that a 5-card poker hand contains exactly three hearts?

16. Calculate P(2000, 3) and $\binom{2000}{3}$.