CSI 30 SPRING 24 PROF. PINEIRO FINAL EXAM PREPARATION

1. Write the propositions using simpler propositions and logical connectives and determine, if possible, true or false.
   (a) If $4 \geq 1$ then $x > 10$.
   (b) If $x < 5$ then $2 \leq 25$
   (c) If $3 \geq -2$ then $4 > 7$.
   (d) If $7 < 1$ then $14 > 10$
2. Show that the compound proposition $(p \wedge (p \to q)) \to q$ is a tautology **using a truth table**.
3. Consider the proposition 'Some people do go good in every situation'.
   (a) Write the proposition using quantifiers. Express clearly the domain of your variables.
   (b) Negate the proposition using quantifiers and express your answer in such a way that no quantifier is immediately preceded by a negation.
4. Let S be the set $A = \{\{1\}, \{2\}, 3\}$. Determine True or False for the following statements:
   (a) $\{2\} \in S$.
   (b) $\{2\} \subset S$.
   (c) $\{3\} \subset S$.
5. Consider the sets $A = \{0, 1, 2\}$ and $B = \{a, b, c\}$.
   (a) List the elements in $\mathcal{P}(B)$.
   (b) Build $A \times B$.
   (c) Give an example of a function from A to B. Is your function one-to-one? onto?
   (d) Give an example of a **relation from** A to B that **is not a function**.
6. (5 points) Suppose that the universe $\mathbb{U} = \{\text{prime numbers} \leq 30\}$, $A = \{7, 13, 29\}$ and $B = \{5, 11, 13, 17, 29\}$.
   (a) Determine $A \cup B$ and $|A \cup B|$.
   (b) Determine $A \cap B$ and $|A \cap B|$.
   (c) Determine $\bar{A}$ and $|\bar{A}|$.
   (d) Determine $A - B$.
   (e) Represent A with a bit string of length 10 using in $\mathbb{U}$ the increasing order.

7. (5 points) Given the algorithm:

   procedure partial($a_1, a_2, a_3, \ldots, a_n$: integers)
   $sum_1 := 0$
   $sum_2 := 0$
   for i := 1 to n
     if ($a_i > 0$): $sum_1 := sum_1 + a_i$
     if ($a_i < 0$): $sum_2 := sum2 + a_i$
   return(sum_1, sum_2)

   For the set of values $\{-4, 5, -7, 2, 9, 0, -2\}$ as input for the above algorithm, what are values of $prod_1$ and $prod_2$ that will be returned?

8. Find the greatest common divisor GCD$(578, 153)$ using the Euclidean Algorithm:

   procedure GCD (a, b: positive integers):
   x := a
   y := b
   While $y \neq 0$:
     r := x mod y
     x := y
     y := r
   return('The GCD is' : x)
   Find integers t, s such that $315t + 140s = \text{GCD}(315, 140)$.

9. Consider the numbers $n = 347$ and $m = (1D3C)_{16}$.
   (a) Find the representation of n is base 16.
   (b) Compute the decimal representation of m. (Use $A = 10, B = 11, \ldots, F = 15$).
   (c) Find $m + n$ in base 16.
   (d) Find $m + n$ in base 10.

10. In how many ways can the letters of the word 'AYAYIYO' be arranged?

11. What is the coefficient of x^2y^4 in the expansion of the binomial $(3x - 5y)^6$.

12. How many bit-strings of length 6 contain not consecutive zeroes?

13. How many integers from 1 to 100 are divisible by 2 or by 3?

14. Consider the following generator of pseudo-random numbers:

$$x_n = (5x_{n-1} + 7) \bmod 12, \quad \text{with seed} \quad x_0 = 4.$$

   What sequence of pseudo-random numbers does it generate?

15. What is the probability that a 5-card poker hand contains exactly three hearts?

16. Calculate $P(2000, 3)$ and $\binom{2000}{3}$.